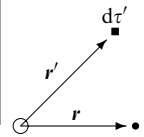


7.2 Static fields

Electrostatics

Electrostatic potential	$\mathbf{E} = -\nabla\phi$	(7.1)	$\mathbf{E}$ electric field ϕ electrostatic potential
Potential difference ^a	$\phi_a - \phi_b = \int_a^b \mathbf{E} \cdot d\mathbf{l} = -\int_b^a \mathbf{E} \cdot d\mathbf{l}$	(7.2)	ϕ_a potential at a ϕ_b potential at b $d\mathbf{l}$ line element
Poisson's Equation (free space)	$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$	(7.3)	ρ charge density ϵ_0 permittivity of free space
Point charge at $\mathbf{r}'$	$\phi(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 \mathbf{r}-\mathbf{r}' }$	(7.4)	q point charge
	$\mathbf{E}(\mathbf{r}) = \frac{q(\mathbf{r}-\mathbf{r}')}{4\pi\epsilon_0 \mathbf{r}-\mathbf{r}' ^3}$	(7.5)	
Field from a charge distribution (free space)	$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{ \mathbf{r}-\mathbf{r}' ^3} d\tau'$	(7.6)	$d\tau'$ volume element $\mathbf{r}'$ position vector of $d\tau'$

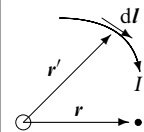
^aBetween points a and b along a path $\mathbf{l}$.



Magnetostatics^a

Magnetic scalar potential	$\mathbf{B} = -\mu_0\nabla\phi_m$	(7.7)	ϕ_m magnetic scalar potential $\mathbf{B}$ magnetic flux density
ϕ_m in terms of the solid angle of a generating current loop	$\phi_m = \frac{I\Omega}{4\pi}$	(7.8)	Ω loop solid angle I current
Biot-Savart law (the field from a line current)	$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int_{\text{line}} \frac{d\mathbf{l} \times (\mathbf{r}-\mathbf{r}')}{ \mathbf{r}-\mathbf{r}' ^3}$	(7.9)	$d\mathbf{l}$ line element in the direction of the current $\mathbf{r}'$ position vector of $d\mathbf{l}$
Ampère's law (differential form)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$	(7.10)	$\mathbf{J}$ current density μ_0 permeability of free space
Ampère's law (integral form)	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{tot}}$	(7.11)	I_{tot} total current through loop

^aIn free space.



Capacitance^a

Of sphere, radius a	$C = 4\pi\epsilon_0\epsilon_r a$	(7.12)
Of circular disk, radius a	$C = 8\epsilon_0\epsilon_r a$	(7.13)
Of two spheres, radius a , in contact	$C = 8\pi\epsilon_0\epsilon_r a \ln 2$	(7.14)
Of circular solid cylinder, radius a , length l	$C \simeq [8 + 4.1(l/a)^{0.76}] \epsilon_0\epsilon_r a$	(7.15)
Of nearly spherical surface, area S	$C \simeq 3.139 \times 10^{-11} \epsilon_r S^{1/2}$	(7.16)
Of cube, side a	$C \simeq 7.283 \times 10^{-11} \epsilon_r a$	(7.17)
Between concentric spheres, radii $a < b$	$C = 4\pi\epsilon_0\epsilon_r ab(b-a)^{-1}$	(7.18)
Between coaxial cylinders, radii $a < b$	$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}$ per unit length	(7.19)
Between parallel cylinders, separation $2d$, radii a	$C = \frac{\pi\epsilon_0\epsilon_r}{\operatorname{arcosh}(d/a)}$ per unit length	(7.20)
	$\simeq \frac{\pi\epsilon_0\epsilon_r}{\ln(2d/a)} \quad (d \gg a)$	(7.21)
Between parallel, coaxial circular disks, separation d , radii a	$C \simeq \frac{\epsilon_0\epsilon_r\pi a^2}{d} + \epsilon_0\epsilon_r a [\ln(16\pi a/d) - 1]$	(7.22)

^aFor conductors, in an embedding medium of relative permittivity ϵ_r .

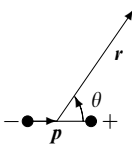
Inductance^a

Of N -turn solenoid (straight or toroidal), length l , area A ($\ll l^2$)	$L = \mu_0 N^2 A / l$	(7.23)
Of coaxial cylindrical tubes, radii a, b ($a < b$)	$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$ per unit length	(7.24)
Of parallel wires, radii a , separation $2d$	$L \simeq \frac{\mu_0}{\pi} \ln \frac{2d}{a}$ per unit length, ($2d \gg a$)	(7.25)
Of wire of radius a bent in a loop of radius $b \gg a$	$L \simeq \mu_0 b \left(\ln \frac{8b}{a} - 2 \right)$	(7.26)

^aFor currents confined to the surfaces of perfect conductors in free space.

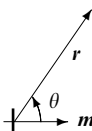
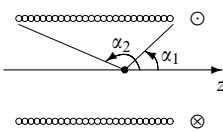


Electric fields^a

Uniformly charged sphere, radius a , charge q	$E(r) = \begin{cases} \frac{q}{4\pi\epsilon_0 a^3} \mathbf{r} & (r < a) \\ \frac{q}{4\pi\epsilon_0 r^3} \mathbf{r} & (r \geq a) \end{cases} \quad (7.27)$	
Uniformly charged disk, radius a , charge q (on axis, z)	$E(z) = \frac{q}{2\pi\epsilon_0 a^2} z \left(\frac{1}{ z } - \frac{1}{\sqrt{z^2 + a^2}} \right) \quad (7.28)$	
Line charge, charge density λ per unit length	$E(r) = \frac{\lambda}{2\pi\epsilon_0 r^2} \mathbf{r} \quad (7.29)$	
Electric dipole, moment $\mathbf{p}$ (spherical polar coordinates, θ angle between $\mathbf{p}$ and $\mathbf{r}$)	$E_r = \frac{p \cos \theta}{2\pi\epsilon_0 r^3} \quad (7.30)$	
	$E_\theta = \frac{p \sin \theta}{4\pi\epsilon_0 r^3} \quad (7.31)$	
Charge sheet, surface density σ	$E = \frac{\sigma}{2\epsilon_0} \quad (7.32)$	

^aFor $\epsilon_r = 1$ in the surrounding medium.

Magnetic fields^a

Uniform infinite solenoid, current I , n turns per unit length	$B = \begin{cases} \mu_0 n I & \text{inside (axial)} \\ 0 & \text{outside} \end{cases} \quad (7.33)$	
Uniform cylinder of current I , radius a	$B(r) = \begin{cases} \mu_0 I r / (2\pi a^2) & r < a \\ \mu_0 I / (2\pi r) & r \geq a \end{cases} \quad (7.34)$	
Magnetic dipole, moment $\mathbf{m}$ (θ angle between $\mathbf{m}$ and $\mathbf{r}$)	$B_r = \mu_0 \frac{m \cos \theta}{2\pi r^3} \quad (7.35)$	
	$B_\theta = \frac{\mu_0 m \sin \theta}{4\pi r^3} \quad (7.36)$	
Circular current loop of N turns, radius a , along axis, z	$B(z) = \frac{\mu_0 N I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}} \quad (7.37)$	
The axis, z , of a straight solenoid, n turns per unit length, current I	$B_{\text{axis}} = \frac{\mu_0 n I}{2} (\cos \alpha_1 - \cos \alpha_2) \quad (7.38)$	

^aFor $\mu_r = 1$ in the surrounding medium.

Image charges

Real charge, $+q$, at a distance:	image point	image charge
b from a conducting plane	$-b$	$-q$
b from a conducting sphere, radius a	a^2/b	$-qa/b$
b from a plane dielectric boundary:		
seen from free space	$-b$	$-q(\epsilon_r - 1)/(\epsilon_r + 1)$
seen from the dielectric	b	$+2q/(\epsilon_r + 1)$

